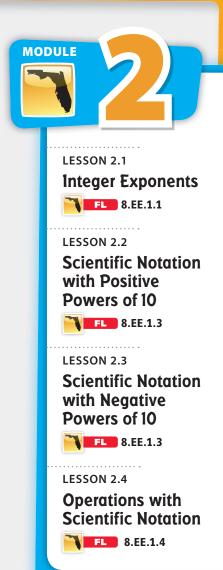
## **Exponents and Scientific Notation**



## ESSENTIAL QUESTION

How can you use scientific notation to solve real-world problems?



The distance from Earth to other planets, moons, and stars is a very great number of kilometers. To make it easier to write very large and very small numbers, we use scientific notation.





my.hrw.com

Go digital with your write-in student edition, accessible on any device.



my.hrw.com

#### Math On the Spot

Scan with your smart phone to jump directly to the online edition, video tutor, and more.



#### **Animated Math**

Interactively explore key concepts to see how math works.



#### **Personal Math Trainer**

Get immediate feedback and help as you work through practice sets.

# Are YOU Ready?

Complete these exercises to review skills you will need for this module.



## **Exponents**

**EXAMPLE**  $10^4 = 10 \times 10 \times 10 \times 10$ = 10,000 Write the exponential expression as a product. Simplify.

#### Write each exponential expression as a decimal.

**1.** 10<sup>2</sup> \_\_\_\_\_ **2.** 10<sup>3</sup> \_\_\_\_\_ **3.** 10<sup>5</sup> \_\_\_\_\_ **4.** 10<sup>7</sup> \_\_\_\_\_

## Multiply and Divide by Powers of 10

EXAMPLE	$0.0478 \times 10^5 = 0.0478 \times 100,000$ = 4,780	Identify the number of zeros in the power of 10. When multiplying, move the decimal point to the right the same number of places as the number of zeros.
	$37.9 \div 10^4 = 37.9 \div 10,000 \\= 0.00379$	Identify the number of zeros in the power of 10. When dividing, move the decimal point to the <i>left</i> the same number of places as the number of zeros.

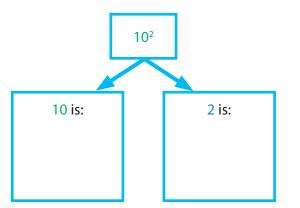
#### Find each product or quotient.

5.	45.3 × 10 <sup>3</sup>	<b>6.</b> 7.08÷	10 <sup>2</sup> <b>7.</b>	$0.00235 \times 10^{6}$	8.	3,600 ÷ 10 <sup>4</sup>
9.	0.5 × 10 <sup>2</sup>	<b>10.</b> 67.7 ÷	10 <sup>5</sup> <b>11.</b>	0.0057 × 10 <sup>4</sup>	12.	195÷10 <sup>6</sup>

**Reading** Start-Up

## **Visualize Vocabulary**

Use the ✔ words to complete the Venn diagram. You can put more than one word in each section of the diagram.



## **Understand Vocabulary**

#### Complete the sentences using the preview words.

- 1. A number produced by raising a base to an exponent
  - is a \_\_\_\_\_\_.
- **2.** \_\_\_\_\_\_ is a method of writing very large or very small numbers by using powers of 10.
- **3.** A \_\_\_\_\_\_ is any number that can be expressed as a ratio of two integers.

## **Active Reading**

**Two-Panel Flip Chart** Create a two-panel flip chart to help you understand the concepts in this module. Label one flap "Positive Powers of 10" and the other flap "Negative Powers of 10." As you study each lesson, write important ideas under the appropriate flap. Include sample problems that will help you remember the concepts later when you look back at your notes.

## Vocabulary

#### **Review Words**

- ✓ base (base)
- exponent (exponente) integers (enteros)
- positive number (número positivo)

standard notation (notación estándar)

#### **Preview Words**

power (potencia) rational number (número racional) real numbers (número real) scientific notation (notación científica) whole number (número entero)



## MODULE 2 Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

## **8.EE.1.1**

Know and apply the properties of integer exponents to generate equivalent numerical expressions.

#### **Key Vocabulary**

#### integer (entero)

The set of whole numbers and their opposites

#### exponent (exponente)

The number that indicates how many times the base is used as a factor.

## What It Means to You

You will use the properties of integer exponents to find equivalent expressions.

#### **UNPACKING EXAMPLE 8.EE.1.1**

Evaluate two different ways.

 $\frac{8^3}{8^5} \qquad \frac{8^3}{8^5} = \frac{8 \cdot 8 \cdot 8}{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8} = \frac{1}{8 \cdot 8} = \frac{1}{64}$  $\frac{8^3}{8^5} = 8^{(3-5)} = 8^{-2} = \frac{1}{8^2} = \frac{1}{8 \cdot 8} = \frac{1}{64}$  $(3^2)^4 \qquad (3^2)^4 = (3^2)(3^2)(3^2)(3^2) = 3^{2+2+2+2} = 3^8 = 6,561$  $(3^2)^4 = 3^{(2-4)} = 3^8 = 6,561$ 

## **₹**∎ 8.EE.1.3

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

### **Key Vocabulary**

#### scientific notation (notación científica)

A method of writing very large or very small numbers by using powers of 10.



Visit my.hrw.com to see all Florida Math Standards unpacked.

🕑 my.hrw.com

C Houghton Mifflin Harcourt Publishing Company

## What It Means to You

You will convert very large numbers to scientific notation.

## UNPACKING EXAMPLE 8.EE.1.3

There are about 55,000,000,000 cells in an average-sized adult. Write this number in scientific notation.

Move the decimal point to the left until you have a number that is greater than or equal to 1 and less than 10.

5.50000000Move the decimal point 10 places to the left.5.5Remove the extra zeros.

You would have to multiply 5.5 by  $10^{10}$  to get 55,000,000,000.

 $55,000,000,000 = 5.5 \times 10^{10}$ 

## LESSON 2.1 Integer Exponents ESSENTIAL QUESTION

How can you develop and use the properties of integer exponents?

EXPLORE ACTIVITY 1

FL 8.EE.1.1

## **Using Patterns of Integer Exponents**

The table below shows powers of 5, 4, and 3.

5 <sup>4</sup> = 625	$5^3 = 125$	$5^2 = 25$	$5^1 = 5$	5 <sup>0</sup> =	5 <sup>-1</sup> =	5 <sup>-2</sup> =
$4^4 = 256$	$4^3 = 64$	$4^2 = 16$	$4^1 = 4$	4 <sup>0</sup> =	4 <sup>-1</sup> =	4 <sup>-2</sup> =
3 <sup>4</sup> = 81	$3^3 = 27$	$3^2 = 9$	3 <sup>1</sup> = 3	3 <sup>0</sup> =	3 <sup>-1</sup> =	3 <sup>-2</sup> =

What pattern do you see in the powers of 5?

**B** What pattern do you see in the powers of 4?

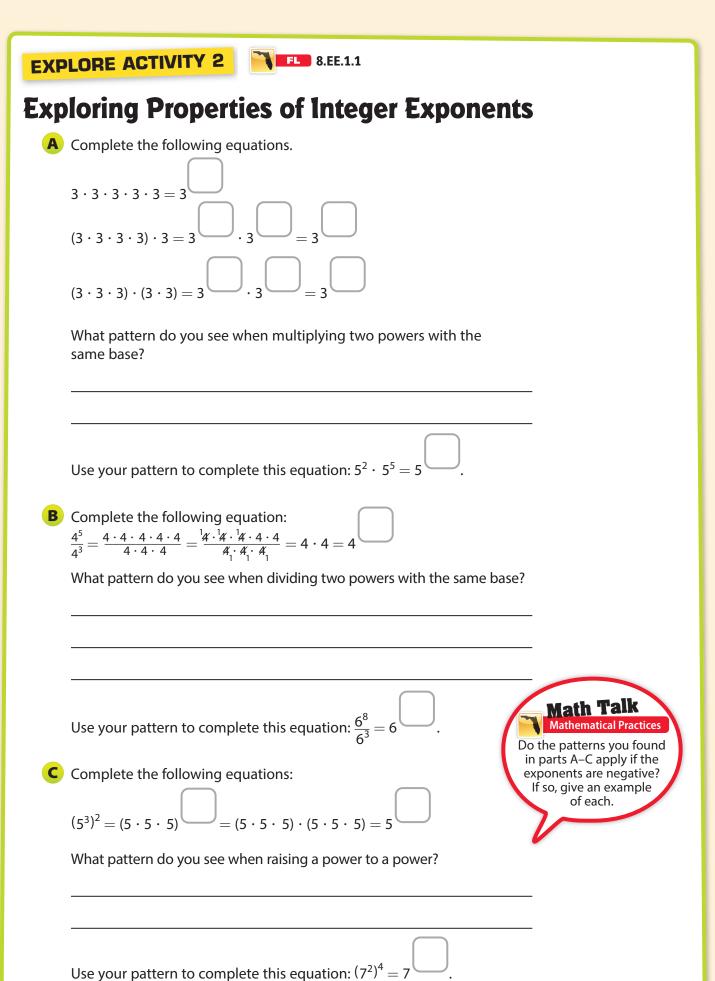
What pattern do you see in the powers of 3?

- **D** Complete the table for the values of  $5^0$ ,  $5^{-1}$ ,  $5^{-2}$ .
- **E** Complete the table for the values of  $4^0$ ,  $4^{-1}$ ,  $4^{-2}$ .
- **F** Complete the table for the values of  $3^0$ ,  $3^{-1}$ ,  $3^{-2}$ .

## Reflect

- **1.** Make a Conjecture Write a general rule for the value of  $a^0$ .
- **2.** Make a Conjecture Write a general rule for the value of  $a^{-n}$ .

C Houghton Mifflin Harcourt Publishing Company



C Houghton Mifflin Harcourt Publishing Company

## Reflect

Let *m* and *n* be integers.

- **3.** Make a Conjecture Write a general rule for the value of  $a^m \cdot a^n$ .
- **4.** Make a Conjecture Write a general rule for the value of  $\frac{a^m}{a^n}$ , a = 0.
- **5.** Make a Conjecture Write a general rule for the value of  $(a^m)^n$ .

## **Applying Properties of Integer Exponents**

You can use the general rules you found in the Explore Activities to simplify more complicated expressions.

## **EXAMPLE 1**

#### Simplify each expression.

<b>A</b> $(5-2)^5 \cdot 3^{-8} + (5+$	2) <sup>0</sup>
$(3)^5 \cdot 3^{-8} + (7)^0$	Simplify within parentheses.
$3^{5+(-8)}+1$	Use properties of exponents.
$3^{-3} + 1$	Simplify.
$\frac{1}{27} + 1 = 1\frac{1}{27}$	Apply the rule for negative exponents and add.

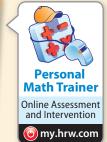
<b>B</b> $\frac{\left[(3+1)^2\right]^3}{(7-3)^2}$	
$\frac{(4^2)^3}{4^2}$	Simplify within parentheses.
$\frac{4^6}{4^2}$	Use properties of exponents.
4 <sup>6-2</sup>	Use properties of exponents.
$4^4 = 256$	Simplify.

YOUR TURN

Simplify each expression.

6.  $\frac{[(6-1)^2]^2}{(3+2)^3}$ 

C Houghton Mifflin Harcourt Publishing Company



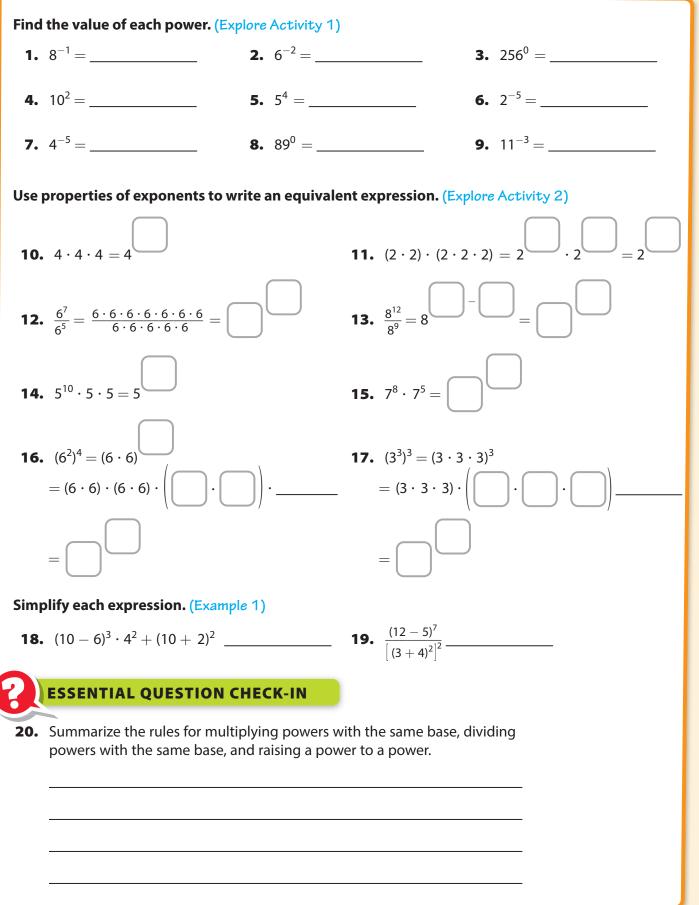
Math On the Spot

My Notes

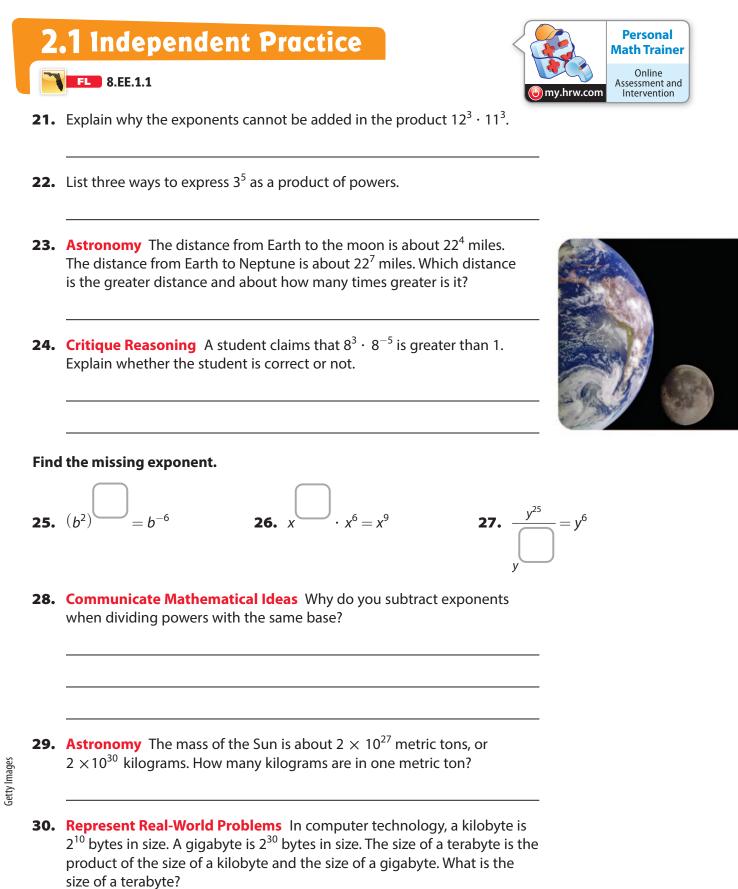
8.EE.1.1

**7.**  $(2^2)^3 - (10 - 6)^3 \cdot 4^{-5}$ 

## **Guided Practice**



O Houghton Mifflin Harcourt Publishing Company 
 I hage Credits: 
 OJupiterimages/



37

**31.** Write equivalent expressions for  $x^7 \cdot x^{-2}$  and  $\frac{x^7}{x^2}$ . What do you notice? Explain how your results relate to the properties of integer exponents.

A toy store is creating a large window display of different colored cubes stacked in a triangle shape. The table shows the number of cubes in each row of the triangle, starting with the top row.

Row	1	2	3	4
Number of cubes in each row	3	3 <sup>2</sup>	3 <sup>3</sup>	3 <sup>4</sup>

- **32.** Look for a Pattern Describe any pattern you see in the table.
- **33.** Using exponents, how many cubes will be in Row 6? How many times as many cubes will be in Row 6 than in Row 3?
- **34.** Justify Reasoning If there are 6 rows in the triangle, what is the total number of cubes in the triangle? Explain how you found your answer.

FOCUS ON HIGHER ORDER THINKING

- **35.** Critique Reasoning A student simplified the expression  $\frac{6^2}{36^2}$  as  $\frac{1}{3}$ . Do you agree with this student? Explain why or why not.
- **36.** Draw Conclusions Evaluate  $-a^n$  when a = 3 and n = 2, 3, 4, and 5. Now evaluate  $(-a)^n$  when a = 3 and n = 2, 3, 4, and 5. Based on this sample, does it appear that  $-a^n = (-a)^n$ ? If not, state the relationships, if any, between  $-a^n$  and  $(-a)^n$ .

**37.** Persevere in Problem Solving A number to the 12th power divided by the same number to the 9th power equals 125. What is the number?

Work Area

# Scientific Notation with Positive Powers of 10

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, ....

FL 8.EE.1.3

**ESSENTIAL QUESTION** 

How can you use scientific notation to express very large quantities?

EXPLORE ACTIVITY



## **Using Scientific Notation**

Scientific notation is a method of expressing very large and very small numbers as a product of a number greater than or equal to 1 and less than 10, and a power of 10.

Real

world

The weights of various sea creatures are shown in the table. Write the weight of the blue whale in scientific notation.

Sea Creature	Blue whale	Gray whale	Whale shark
Weight (lb)	250,000	68,000	41,200

A Move the decimal point in 250,000 to the left as many places as necessary to find a number that is greater than or equal to 1 and less than 10.

What number did you find? \_\_\_\_\_

**B** Divide 250,000 by your answer to **A**. Write your answer as a power of 10.

Combine your answers to A and B to represent 250,000.



Repeat steps \Lambda through C to write the weight of the whale shark in scientific notation.

1,200 =	×	10	

## Reflect

1. How many places to the left did you move the decimal point to write

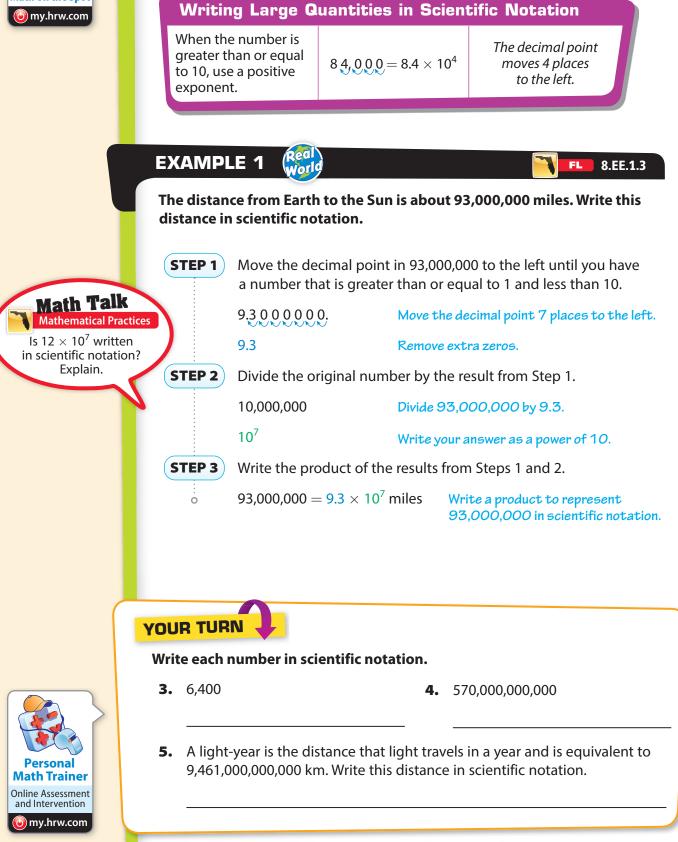
41,200 in scientific notation? \_\_\_\_\_

2. What is the exponent on 10 when you write 41,200 in scientific notation?



## Writing a Number in Scientific Notation

To translate between standard notation and scientific notation, you can count the number of places the decimal point moves.



## Writing a Number in Standard Notation

To translate between scientific notation and standard notation, move the decimal point the number of places indicated by the exponent in the power of 10. When the exponent is positive, move the decimal point to the right and add placeholder zeros as needed.

## **EXAMPLE 2**

STEP 1

### Write 3.5 $\times$ 10<sup>6</sup> in standard notation.

Use the exponent of the power of 10 to see how many places to move the decimal point.

- Place the decimal point. Since you are going to write a number greater than 3.5, move the decimal point to the right. Add placeholder zeros if necessary.
- The number  $3.5 \times 10^6$  written in standard notation is 3,500,000.

## Reflect

ò

**6.** Explain why the exponent in  $3.5 \times 10^6$  is 6, while there are only 5 zeros in 3,500,000.

What is the exponent on 10 when you write 5.3 in scientific notation? 7.

YOUR TURN

**8.** 7.034  $\times$  10<sup>9</sup>

Write each number in standard notation.

10. The mass of one roosting colony of Monarch butterflies in Mexico was estimated at  $5 \times 10^6$  grams. Write this mass in standard notation.

**9.**  $2.36 \times 10^5$ 

**Math Trainer Online Assessment** and Intervention my.hrw.com





My Notes



3500000.

8.EE.1.3



## **Guided Practice**

Write	e each number in scientific notation. (Exp	lore Activity	and Example 1)
1.	58,927 Hint: Move the decimal left 4 places.	2.	1,304,000,000 Hint: Move the decimal left 9 places.
3.	6,730,000	4.	13,300
5.	An ordinary quarter contains about 97,700,000,000,000,000,000,000 atoms.	6.	The distance from Earth to the Moon is about 384,000 kilometers.
Write	e each number in standard notation. (Exa	mple 2)	
7.	$4 \times 10^5$ Hint: Move the decimal right 5 places.	8.	1.8499 $ imes$ 10 <sup>9</sup> Hint: Move the decimal right 9 places.
9.	$6.41 \times 10^{3}$	10.	$8.456 \times 10^{7}$
11.	$8 \times 10^5$	12.	$9 \times 10^{10}$
13.	Diana calculated that she spent about 5.4 $\times$ homework during October. Write this time i		
14.	The town recycled 7.6 $\times$ 10 <sup>6</sup> cans this year standard notation. (Example 2)	. Write the n	umber of cans in
2	ESSENTIAL QUESTION CHECK-IN		
15.	Describe how to write 3,482,000,000 in scie	entific notati	ion.

## **2.2** Independent Practice

## 8.EE.1.3

**Paleontology** Use the table for problems 16-21. Write the estimated weight of each dinosaur in scientific notation.

Estimated Weight of Dinosaurs			
Name	Pounds		
Argentinosaurus	220,000		
Brachiosaurus	100,000		
Apatosaurus	66,000		
Diplodocus	50,000		
Camarasaurus	40,000		
Cetiosauriscus	19,850		

- **16.** Apatosaurus \_\_\_\_\_
- **17.** Argentinosaurus \_\_\_\_\_
- **18.** Brachiosaurus
- **19.** Camarasaurus \_\_\_\_\_
- **20.** Cetiosauriscus
- **21.** Diplodocus
- **22.** A single little brown bat can eat up to 1,000 mosquitoes in a single hour. Express in scientific notation how many mosquitoes a little brown bat might eat in 10.5 hours.
- **23.** Multistep Samuel can type nearly 40 words per minute. Use this information to find the number of hours it would take him to type 2.6  $\times$  10<sup>5</sup> words.





**a.** If you were as strong as this insect, explain how you could find how many pounds you could lift.

- **b.** Complete the calculation to find how much you could lift, in pounds, if you were as strong as an Archegozetes *longisetosus* mite. Express your answer in both scientific notation and standard notation.
- **25.** During a discussion in science class, Sharon learns that at birth an elephant weighs around 230 pounds. In four herds of elephants tracked by conservationists, about 20 calves were born during the summer. In scientific notation, express approximately how much the calves weighed all together.
- 26. Classifying Numbers Which of the following numbers are written in scientific notation?

$0.641 \times 10^{3}$	$9.999 \times 10^{4}$
$2  imes 10^1$	$4.38 \times 5^{10}$

Class.

Personal

Online

- **27.** Explain the Error Polly's parents' car weighs about 3500 pounds. Samantha, Esther, and Polly each wrote the weight of the car in scientific notation. Polly wrote  $35.0 \times 10^2$ , Samantha wrote  $0.35 \times 10^4$ , and Esther wrote  $3.5 \times 10^4$ .
  - a. Which of these girls, if any, is correct?
  - **b.** Explain the mistakes of those who got the question wrong.

**28.** Justify Reasoning If you were a biologist counting very large numbers of cells as part of your research, give several reasons why you might prefer to record your cell counts in scientific notation instead of standard notation.

FOCUS ON HIGHER ORDER THINKING

- **29.** Draw Conclusions Which measurement would be least likely to be written in scientific notation: number of stars in a galaxy, number of grains of sand on a beach, speed of a car, or population of a country? Explain your reasoning.
- **30.** Analyze Relationships Compare the two numbers to find which is greater. Explain how you can compare them without writing them in standard notation first.

 $4.5\times10^6 \qquad 2.1\times10^8$ 

**31.** Communicate Mathematical Ideas To determine whether a number is written in scientific notation, what test can you apply to the first factor, and what test can you apply to the second factor?

Work Area

 $H_{01}$ 

#### **Scientific Notation FL** 8.EE.1.3 **2.3** with Negative Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very **Powers of 10** small quantities, .... **ESSENTIAL QUESTION** How can you use scientific notation to express very small quantities? Real EXPLORE ACTIVITY FL 8.EE.1.3 **Negative Powers of 10** Animated Math You can use what you know about writing very large numbers in scientific my.hrw.com notation to write very small numbers in scientific notation. A typical human hair has a diameter of 0.000025 meter. Write this number in scientific notation. A Notice how the decimal point moves in the list below. Complete the list. $2.345 \times 10^{\circ}$ $2.345 \times 10^{\circ}$ = 2.345 lt moves one 2.3 4 5 It moves one = 2345 place to the right with place to the = 0.2,345 $2.345 \times 10^{-1}$ $2.345 \times 10^{1}$ left with each = 234.5 each increasing $2.345 \times 10^{-2}$ decreasing = 0.0 2 3 4 5 $2.345 \times 10^{2}$ power of 10. power of 10. $2.345 \times 10^{\circ}$ = 2,3,4,5 $2.345 \times 10^{\circ}$ = 0.0 0 2 3 4 5 **B** Move the decimal point in 0.000025 to the right as many places as necessary to find a number that is greater than or equal to 1 and

less than 10. What number did you find? \_\_\_\_\_\_

C Divide 0.000025 by your answer to B.

Write your answer as a power of 10. \_\_\_\_\_

D Combine your answers to B and C to represent 0.000025 in

scientific notation.

## Reflect

- 1. When you move the decimal point, how can you know whether you are increasing or decreasing the number?
- **2.** Explain how the two steps of moving the decimal and multiplying by a power of 10 leave the value of the original number unchanged.



## **Writing a Number in Scientific Notation**

To write a number less than 1 in scientific notation, move the decimal point right and use a negative exponent.

Writing Small Quantities in Scientific Notation				
When the number is between 0 and 1, use a negative exponent.	$0.0783 = 7.83 \times 10^{-2}$	The decimal point moves 2 places to the right.		

## EXAMPLE 1



#### The average size of an atom is about 0.0000003 centimeter across. Write the average size of an atom in scientific notation.

Move the decimal point as many places as necessary to find a number that is greater than or equal to 1 and less than 10.

- **STEP 1** Place the decimal point. 3.0
- **STEP 2** Count the number of places you moved the <u>decimal</u> point. 8
- **STEP 3** Multiply 3.0 times a power of 10.  $3.0 \times 10^{-8}$

Since 0.0000003 is less than 1, you moved the decimal point to the right and the exponent on 10 is negative.

The average size of an atom in scientific notation is 3.0 imes 10<sup>-8</sup>.

## Reflect

ò

**3. Critical Thinking** When you write a number that is less than 1 in scientific notation, how does the power of 10 differ from when you write a number greater than 1 in scientific notation?

## 

#### Write each number in scientific notation.

**4.** 0.0000829

- **5.** 0.00000302
- **6.** A typical red blood cell in human blood has a diameter of approximately 0.000007 meter. Write this diameter

in scientific notation.



Personal

Math Trainer Online Assessment

and Intervention omy.hrw.com

## Writing a Number in Standard Notation

To translate between scientific notation and standard notation with very small numbers, you can move the decimal point the number of places indicated by the exponent on the power of 10. When the exponent is negative, move the decimal point to the left.



Math Talk Mathematical Practices

Describe the two factors

that multiply together to form a number written in scientific notation.

FL.

8.EE.1.3

## EXAMPLE 2

Platelets are one component of human blood. A typical platelet has a diameter of approximately 2.33  $\times$  10<sup>-6</sup> meter. Write 2.33  $\times$  10<sup>-6</sup> in standard notation.

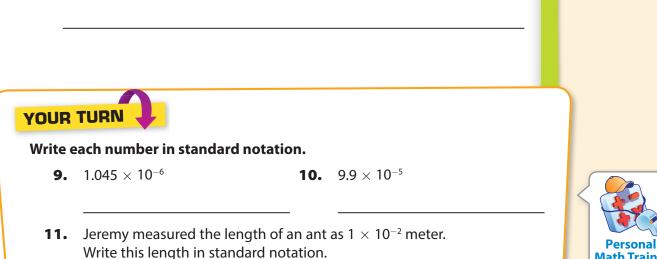
- **STEP 1** Use the exponent of the power of 10 to see 6 places how many places to move the decimal point.
- **STEP 2** Place the decimal point. Since you are going to 0.0000233 write a number less than 2.33, move the decimal point to the *left*. Add placeholder zeros if necessary.
  - The number 2.33  $\times$  10<sup>-6</sup> in standard notation is 0.00000233.

## Reflect

ò

**7.** Justify Reasoning Explain whether  $0.9 \times 10^{-5}$  is written in scientific notation. If not, write the number correctly in scientific notation.

**8.** Which number is larger,  $2 \times 10^{-3}$  or  $3 \times 10^{-2}$ ? Explain.



and Intervention

Math Trainer Online Assessment

## **Guided Practice**

Write	e each number in scientific notation. (Exp	lore Activity	( and Example 1)		
1.	0.000487 Hint: Move the decimal right 4 places.	2.	0.000028 Hint: Move the decimal right 5 places.		
3.	0.000059	4.	0.0417		
5.	Picoplankton can be as small as 0.00002 centimeter.	6.	The average mass of a grain of sand on a beach is about 0.000015 gram.		
Write	e each number in standard notation. (Exa	mple 2)			
7.	$2\times 10^{-5}$ Hint: Move the decimal left 5 places.	8.	$3.582 \times 10^{-6}$ Hint: Move the decimal left 6 places.		
9.	$8.3  imes 10^{-4}$	10.	2.97 × 10 <sup>-2</sup>		
11.	9.06 × 10 <sup>-5</sup>	12.	$4  imes 10^{-5}$		
13.	The average length of a dust mite is appro Write this number in scientific notation. (E	•	001 meter.		
14.	<b>4.</b> The mass of a proton is about $1.7 \times 10^{-24}$ gram. Write this number in standard notation. (Example 2)				
2)	ESSENTIAL QUESTION CHECK-IN				
15.	Describe how to write 0.0000672 in scienti	fic notation.			

## **2.3** Independent Practice

Class.

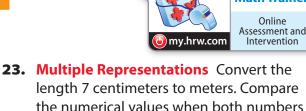
## 8.EE.1.3

Use the table for problems 16-21. Write the diameter of the fibers in scientific notation.

Average Diameter of Natural Fibers			
Animal	Fiber Diameter (cm)		
Vicuña	0.0008		
Angora rabbit	0.0013		
Alpaca	0.00277		
Angora goat	0.0045		
Llama	0.0035		
Orb web spider	0.015		

#### **16.** Alpaca

- **17.** Angora rabbit
- **18.** Llama
- **19.** Angora goat
- **20.** Orb web spider
- **21.** Vicuña
- **22.** Make a Conjecture Which measurement would be least likely to be written in scientific notation: the thickness of a dog hair, the radius of a period on this page, the ounces in a cup of milk? Explain your reasoning.



are written in scientific notation.

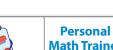
Date\_

**24. Draw Conclusions** A graphing calculator displays  $1.89 \times 10^{12}$  as  $1.89_{E}12$ . How do you think it would display 1.89  $\times$  10<sup>-12</sup>? What

does the  $\varepsilon$  stand for?

25. Communicate Mathematical Ideas When a number is written in scientific notation, how can you tell right away whether or not it is greater than or equal to 1?

- **26.** The volume of a drop of a certain liquid is 0.000047 liter. Write the volume of the drop of liquid in scientific notation.
- **27. Justify Reasoning** If you were asked to express the weight in ounces of a ladybug in scientific notation, would the exponent of the 10 be positive or negative? Justify your response.



**Math Trainer** Online Assessment and Intervention

## **Physical Science** The table shows the length of the radii of several very small or very large items. Complete the table.

	ltem	Radius in Meters (Standard Notation)	Radius in Meters (Scientific Notation)
28.	The Moon	1,740,000	
29.	Atom of silver		$1.25 imes10^{-10}$
30.	Atlantic wolfish egg	0.0028	
31.	Jupiter		$7.149  imes 10^7$
32.	Atom of aluminum	0.00000000182	
33.	Mars		$3.397 imes10^6$



**34.** List the items in the table in order from the smallest to the largest.

FOCUS ON HIGHER ORDER THINKING

- **35.** Analyze Relationships Write the following diameters from least to greatest.  $1.5 \times 10^{-2}$  m  $1.2 \times 10^{2}$  m  $5.85 \times 10^{-3}$  m  $2.3 \times 10^{-2}$  m  $9.6 \times 10^{-1}$  m
- **36.** Critique Reasoning Jerod's friend Al had the following homework problem:

Express 5.6  $\times$  10<sup>-7</sup> in standard form.

Al wrote 56,000,000. How can Jerod explain Al's error and how to correct it?

**37.** Make a Conjecture Two numbers are written in scientific notation. The number with a positive exponent is divided by the number with a negative exponent. Describe the result. Explain your answer. Work Area

HOT

# **2.4 Operations with 2.4 Scientific Notation**

Perform operations . . . in scientific notation. ... choose units of appropriate size for measurements .... Interpret scientific notation ... generated by technology.

EL 8.EE.1.4

**ESSENTIAL QUESTION** 

How do you add, subtract, multiply, and divide using scientific notation?

## **Adding and Subtracting with Scientific Notation**

Numbers in scientific notation can be added and subtracted, either directly or by rewriting them in standard form.



Red **EXAMPLE 1** 

#### 8.EE.1.4

The table below shows the population of the three largest countries in North America in 2011. Find the total population of these countries.

Country	United States	Canada	Mexico
Population	$3.1 imes10^8$	$3.38  imes 10^7$	$1.1  imes 10^8$

#### Method 1:

STEP 1 First, write each population with the same power of 10.

United States:  $3.1 \times 10^8$ 

 $0.338 \times 10^{8}$ Canada:

 $1.1 \times 10^{8}$ Mexico:

**STEP 2** 

Add the multipliers for each population.

3.1 + 0.338 + 1.1 = 4.538

Write the final answer in scientific notation:  $4.538 \times 10^8$ . **STEP 3** 

#### Method 2:

STEP 1	First, write each number in standard notation.		
	United States:	310,000,000	
- - - - - -	Canada:	33,800,000	

Mexico: 110,000,000

**STEP 2** 

310,000,000 + 33,800,000 + 110,000,000 = 453,800,000

Find the sum of the numbers in standard notation.

**STEP 3** Write the answer in scientific notation:  $4.538 \times 10^8$ .







Mathematical Practices Could you write

 $2.025 \times 10^{14}$  in standard notation to do the division?

Would this be a good way to solve the problem?



1. Using the population table above, how many more people live in Mexico than in Canada? Write your answer in scientific notation.

# Multiplying and Dividing with Scientific Notation

Numbers in scientific notation can be multiplied and divided directly by using properties of exponents.

#### EXAMPLE 2 Problem Solving

When the Sun makes an orbit around the center of the Milky Way, it travels  $2.025 \times 10^{14}$  kilometers. The orbit takes 225 million years. At what rate does the Sun travel? Write your answer in scientific notation.

## Analyze Information

The answer is the number of kilometers per year that the Sun travels around the Milky Way.

## Formulate a Plan

Set up a division problem using Rate  $=\frac{\text{Distance}}{\text{Time}}$  to represent the situation.

## Solve

- **STEP 1** Substitute the values from the problem into the Rate formula.
  - $\mathsf{Rate} = \frac{2.025 \times 10^{14} \, \mathsf{kilometers}}{225,000,000 \, \mathsf{years}}$
- **STEP 2** Write the expression for rate with years in scientific notation.

$Rate = \frac{2.025 \times 10^{14} \text{ kilometers}}{2.025 \times 10^{14} \text{ kilometers}}$	225 million = $2.25 \times 10^8$
$2.25 \times 10^8$ years	$225$ minimum $= 2.25 \times 10$

**STEP 3** Find the quotient by dividing the decimals and using the laws of exponents.

 $2.025 \div 2.25 = 0.9$ Divide the multipliers. $\frac{10^{14}}{10^8} = 10^{14-8} = 10^6$ Divide the powers of 10.

Combine the answers to write the rate in scientific notation.

Rate =  $0.9 \times 10^6 = 9.0 \times 10^5$  km per year

## Justify and Evaluate

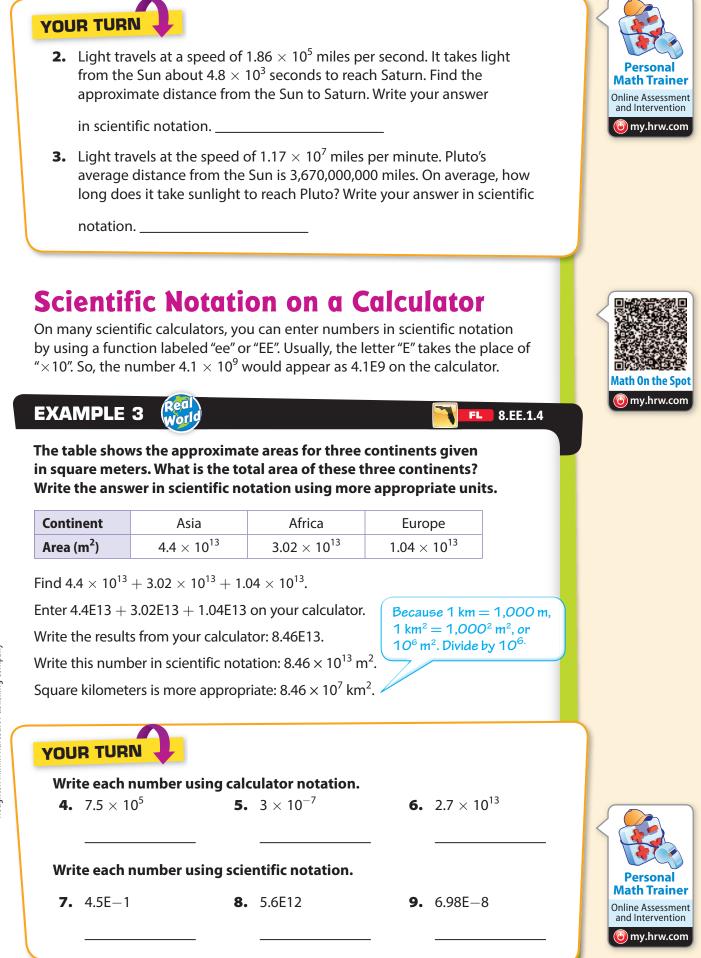
**STEP 4** 

ò

Check your answer using multiplication.

900,000  $\times$  225,000,000 = 202,500,000,000,000, or 2.025  $\times$   $10^{14}.$  The answer is correct.

8.EE.1.4



## **Guided Practice**

Add or subtract. Write your answer in scientific notation. (Example 1)					
1.	$4.2 \times 10^6 + 2.25 \times 10^5 + 2.8 \times 10^6$	2.	$8.5 \times 10^3 - 5.3 \times 10^3 - 1.0 \times 10^2$		
	$4.2 \times 10^{6} + \times 10^{6} + 4.2 + + + + + + + + + + + + + + + + + + +$	- 2.8 × 10 <sup>6</sup>	$8.5 \times 10^3 - 5.3 \times 10^3 -$	) × 10	
3.	$1.25 \times 10^{2} + 0.50 \times 10^{2} + 3.25 \times 10^{2}$	0 <sup>2</sup> <b>4.</b>	$6.2 \times 10^{5} - 2.6 \times 10^{4} - 1.9 \times 10^{2}$		
Mult	iply or divide. Write your answer in	n scientific not	ation. (Example 2)		
5.	$(1.8 \times 10^9)(6.7 \times 10^{12})$	6.	$\frac{3.46 \times 10^{17}}{2 \times 10^9}$		
7.	$(5 \times 10^{12})(3.38 \times 10^6)$		$\frac{8.4 \times 10^{21}}{4.2 \times 10^{14}}$		
	e each number using calculator not				
9.	$3.6 \times 10^{11}$ <b>10.</b> 7	$7.25  imes 10^{-5}$	<b>11.</b> $8 \times 10^{-1}$		
Write	e each number using scientific nota	ation. (Example	:3)		
		1.2E16	<b>14.</b> 9E1		
2)	ESSENTIAL QUESTION CHECK	K-IN			
15.	How do you add, subtract, multiply, scientific notation?	and divide nur	mbers written in		

## **2.4** Independent Practice

## **EL** 8.EE.1.4

- **16.** An adult blue whale can eat  $4.0 \times 10^7$  krill in a day. At that rate, how many krill can an adult blue whale eat in 3.65  $\times$  10<sup>2</sup> days?
- **17.** A newborn baby has about 26,000,000,000 cells. An adult has about  $4.94 \times 10^{13}$  cells. How many times as many cells does an adult have than a newborn? Write your answer in scientific notation.

**Represent Real-World Problems** The table shows the number of tons of waste generated and recovered (recycled) in 2010.



	Paper	Glass	Plastics
Tons generated	7.131 × 10 <sup>7</sup>	$1.153  imes 10^7$	$\textbf{3.104}\times\textbf{10}^{7}$
Tons recovered	$4.457  imes 10^7$	$0.313  imes 10^7$	$0.255  imes 10^7$

- **18.** What is the total amount of paper, glass, and plastic waste generated?
- **19.** What is the total amount of paper, glass, and plastic waste recovered?



Date.

- **Math Trainer** Online Assessment and Intervention
- **20.** What is the total amount of paper, glass, and plastic waste not recovered?

Class

**21.** Which type of waste has the lowest recovery ratio?

#### Social Studies The table shows the approximate populations of three countries.

Country	China	France	Australia
Population	$1.3  imes 10^9$	$\textbf{6.48}\times \textbf{10}^{7}$	$\textbf{2.15}\times\textbf{10}^{7}$

- **22.** How many more people live in France than in Australia?
- **23.** The area of Australia is  $2.95 \times 10^6$  square miles. What is the approximate average number of people per square mile in Australia?
- **24.** How many times greater is the population of China than the population of France? Write your answer in standard notation.
- **25.** Mia is 7.01568  $\times$  10<sup>6</sup> minutes old. Convert her age to more appropriate units using years, months, and days. Assume each month to have 30.5 days.

- **26.** Courtney takes  $2.4 \times 10^4$  steps during her a long-distance run. Each step covers an average of 810 mm. What total distance (in mm) did Courtney cover during her run? Write your answer in scientific notation. Then convert the distance to the more appropriate unit kilometers. Write that answer in standard form.
- **27.** Social Studies The U.S. public debt as of October 2010 was  $\$9.06 \times 10^{12}$ . What was the average U.S. public debt per American if the population in 2010 was  $3.08 \times 10^8$  people?



**28.** Communicate Mathematical Ideas How is multiplying and dividing numbers in scientific notation different from adding and subtracting numbers in scientific notation?

Work Area

- **29.** Explain the Error A student found the product of  $8 \times 10^6$  and  $5 \times 10^9$  to be  $4 \times 10^{15}$ . What is the error? What is the correct product?
- **30.** Communicate Mathematical Ideas Describe a procedure that can be used to simplify  $\frac{(4.87 \times 10^{12}) (7 \times 10^{10})}{(3 \times 10^7) + (6.1 \times 10^8)}$ . Write the expression in scientific notation in simplified form.

nd the value of each power.		(i) my
<b>1.</b> 3 <sup>-4</sup>	<b>2.</b> 35 <sup>°</sup>	<b>3.</b> 4 <sup>4</sup>
se the properties of exponent	-	-
<b>4.</b> 8 <sup>3</sup> · 8 <sup>7</sup>	<b>5.</b> $\frac{12^{\circ}}{12^{2}}$	<b>6.</b> (10 <sup>3</sup> ) <sup>5</sup>
.2 Scientific Notation	with Positive Po	owers of 10
onvert each number to scient	ific notation or stand	dard notation.
7. 2,000	8.	91,007,500
<b>9.</b> 1.0395 × 10 <sup>9</sup>	10.	4 × 10 <sup>2</sup>
<ul> <li>13. 8.9 × 10<sup>-5</sup></li></ul>	entific Notation	
<b>15.</b> 7 × 10 <sup>6</sup> – 5.3 × 10 <sup>6</sup>		$3.4 \times 10^4 + 7.1 \times 10^5$
<b>17.</b> (2 × 10 <sup>4</sup> )(5.4 × 10 <sup>6</sup> )	18.	$\frac{7.86 \times 10^9}{3 \times 10^4} - $
distance from the Sun	is 5.791 $ imes$ 10 $^{7}$ km. Abo	4.503 $\times$ 10 <sup>9</sup> km. Mercury's average out how many times farther from r answer in scientific notation.
ESSENTIAL QUESTION		
	on used in the real wo	



MODULE 2 MIXED REVIEW Assessment Readiness



## **Selected Response**

**1.** Which of the following is equivalent to  $6^{-3}$ ?

(A) 216 (C)  $-\frac{1}{216}$ (B)  $\frac{1}{216}$  (D) -216

- **2.** About 786,700,000 passengers traveled by plane in the United States in 2010. What is this number written in scientific notation?
  - (A) 7,867  $\times$  10<sup>5</sup> passengers
  - (B)  $7.867 \times 10^2$  passengers
  - $\bigcirc$  7.867  $\times$  10<sup>8</sup> passengers
  - (D)  $7.867 \times 10^9$  passengers
- **3.** In 2011, the population of Mali was about  $1.584 \times 10^7$  people. What is this number written in standard notation?
  - (A) 1.584 people
  - (B) 1,584 people
  - © 15,840,000 people
  - D 158,400,000 people
- **4.** The square root of a number is between 7 and 8. Which could be the number?
  - (A) 72 (C) 51
  - **B** 83 **D** 66
- **5.** Each entry-level account executive in a large company makes an annual salary of  $$3.48 \times 10^4$ . If there are  $5.2 \times 10^2$  account executives in the company, how much do they make in all?
  - (A)  $$6.69 \times 10^{1}$
  - **B** \$3.428 × 10<sup>4</sup>
  - € \$3.532 × 10<sup>4</sup>
  - **D** \$1.8096 × 10<sup>7</sup>

**6.** Place the numbers in order from least to greatest.

 $0.24, 4 imes 10^{-2}, 0.042, 2 imes 10^{-4}, 0.004$ 

(A)  $2 \times 10^{-4}$ ,  $4 \times 10^{-2}$ , 0.004, 0.042, 0.24

- **(B)** 0.004, 2 × 10<sup>-4</sup>, 0.042, 4 × 10<sup>-2</sup>, 0.24
- (c) 0.004,  $2 \times 10^{-4}$ ,  $4 \times 10^{-2}$ , 0.042, 0.24
- (D)  $2 \times 10^{-4}$ , 0.004,  $4 \times 10^{-2}$ , 0.042, 0.24
- 7. Guillermo is  $5\frac{5}{6}$  feet tall. What is this number of feet written as a decimal?
  - (A) 5.7 feet (C) 5.83 feet
  - (B) 5.7 feet (D) 5.83 feet
- **8.** A human hair has a width of about  $6.5 \times 10^{-5}$  meter. What is this width written in standard notation?
  - (A) 0.00000065 meter
  - (B) 0.0000065 meter
  - © 0.000065 meter
  - (D) 0.00065 meter

## Mini-Task

- **9.** Consider the following numbers: 7000, 700, 70, 0.7, 0.07, 0.007
  - **a.** Write the numbers in scientific notation.
  - **b.** Look for a pattern in the given list and the list in scientific notation. Which numbers are missing from the lists?
  - **c.** Make a conjecture about the missing numbers.

# Study Guide Review



**Real Numbers** 

## **ESSENTIAL QUESTION**

How can you use real numbers to solve real-world problems?

## **EXAMPLE 1**

Write  $0.\overline{81}$  as a fraction in simplest form.

$$x = 0.\overline{81}$$

$$100x = 81.\overline{81}$$

$$-x - 0.\overline{81}$$

$$99x = 81$$

$$x = \frac{81}{99}$$

$$x = \frac{9}{11}$$

## EXAMPLE 2

Solve each equation for *x*.

<b>A</b> $x^2 = 289$	<b>B</b> $x^3 = 1,000$
$x = \pm \sqrt{289}$	$x = \sqrt[3]{1,000}$
$x = \pm 17$	<i>x</i> = 10
The solutions are 17 and $-17$ .	The solution is 10.

## **EXAMPLE 3**

Write all names that apply to each number.

A $5.\overline{4}$  $5.\overline{4}$  is a repeating decimal.rational, real $5.\overline{4}$  is a repeating decimal.B $\frac{8}{4}$ <br/>whole, integer, rational, real $\frac{8}{4} = 2$ 

 $\sqrt{13}$  irrational, real

13 is a whole number that is not a perfect square.

## **Key Vocabulary**

cube root (raiz cúbica) irrational number (número *irracional*) perfect cube (cubo perfecto) perfect square (cuadrado perfecto) principal square root (raíz *cuadrada principal*) rational number (número racional) real number (número real) repeating decimal (decimal periódico) square root (raíz cuadrada) terminating decimal (decimal finito)

#### **EXAMPLE 4**

### Order 6, $2\pi$ , and $\sqrt{38}$ from least to greatest.

 $2\pi$  is approximately equal to 2 imes 3.14, or 6.28.

 $\sqrt{38}$  is approximately 6.15 based on the following reasoning.

$$\sqrt{36} < \sqrt{38} < \sqrt{49}$$
  $6 < \sqrt{38} < 7$   $6.1^2 = 37.21$   $6.2^2 = 38.44$ 

From least to greatest, the numbers are 6,  $\sqrt{38}$ , and  $2\pi$ .

## EXERCISES

Find the two square roots of each number. If the number is not a perfect square, approximate the values to the nearest 0.05. (Lesson 1.1)

1.	16	<b>2.</b> $\frac{4}{25}$		<b>3.</b> 225		
4.	<u>1</u> <u>49</u>	<b>5.</b> $\sqrt{10}$		<b>6.</b> $\sqrt{18}$		
Write	e each decimal as a fracti	on in simplest fo	orm. (Lesson 1.1)			
7.	0.5	<b>8.</b> 0.63		<b>9.</b> 0.214		
Solve	e each equation for x. (Le	sson 1.1)				
10.	$x^2 = 361$	<b>11.</b> <i>x</i> <sup>3</sup> = 1,72	8	<b>12.</b> $x^2 = \frac{49}{121}$		
Write	all names that apply to	each number. <mark>(</mark> L	esson 1.2)			
13.	$\frac{2}{3}$		<b>14.</b> $-\sqrt{100}$			
15.	<u>15</u> 5		<b>16.</b> √21			
Compare. Write <, >, or =. (Lesson 1.3)						
17.	$\sqrt{7}$ + 5 $\bigcirc$ 7 + $\sqrt{5}$	<b>18.</b> $6 + \sqrt{8}$ (	$\sqrt{6} + 8$	<b>19.</b> $\sqrt{4} - 2 \bigcirc 4 - \sqrt{2}$		
Order the numbers from least to greatest. (Lesson 1.3)						
20.	$\sqrt{81}, \frac{72}{7}, 8.9$		<b>21.</b> $\sqrt{7}$ , 2.55, $\frac{7}{3}$			

Houghton Mifflin Harcourt Publishing Company



## **Exponents and Scientific** Notation

#### **ESSENTIAL QUESTION**

How can you use scientific notation to solve real-world problems?

## **EXAMPLE 1**

Write each measurement in scientific notation.

A The diameter of Earth at the equator is approximately 12,700 kilometers. Move the decimal point in 12,700 four places to the left: 1.2 7 0 0.

 $12,700 = 1.27 \times 10^4$ 

**B** The diameter of a human hair is approximately 0.00254 centimeters. Move the decimal point in 0.00254 three places to the right: 0.0 0 2.5 4

 $0.00254 = 2.54 \times 10^{-3}$ 

## **EXAMPLE 2**

Find the quotient:  $\frac{\textbf{2.4}\times 10^7}{\textbf{9.6}\times 10^3}$ Divide the multipliers:  $2.4 \div 9.6 = 0.25$ Divide the powers of ten:  $\frac{10^7}{10^3} = 10^{7-3} = 10^4$ Combine the answers and write the product in scientific notation.

 $0.25 \times 10^4 = 0.25 \times (10 \times 10^3) = (0.25 \times 10) \times 10^3 = 2.5 \times 10^3$ 

## **EXERCISES**

Write each number in scientific notation. (Lessons 2.2, 2.3)

**1.** 25,500,000 **2.** 0.00734

Write each number in standard notation. (Lessons 2.2, 2.3)

**3.**  $5.23 \times 10^4$  **4.**  $1.33 \times 10^{-5}$ 

Simplify each expression. (Lessons 2.1, 2.4)

**5.**  $(9-7)^3 \cdot 5^0 + (8+3)^2$  **6.**  $\frac{(4+2)^2}{[(9-3)^3]^2}$ 

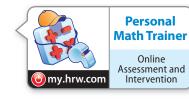
**7.**  $3.2 \times 10^5 + 1.25 \times 10^4 + 2.9 \times 10^5$  **8.**  $(2,600)(3.24 \times 10^4)$ 

## Unit 1 Performance Tasks

- 1. **CAREERS IN MATH** Astronomer An astronomer is studying Proxima Centauri, which is the closest star to our Sun. Proxima Centauri is 39,900,000,000,000,000 meters away.
  - **a.** Write this distance in scientific notation.
  - **b.** Light travels at a speed of  $3.0 \times 10^8$  m/s (meters per second). How can you use this information to calculate the time in seconds it takes for light from Proxima Centauri to reach Earth? How many seconds does it take? Write your answer in scientific notation.
  - **c.** Knowing that 1 year =  $3.1536 \times 10^7$  seconds, how many years does it take for light to travel from Proxima Centauri to Earth? Write your answer in standard notation. Round your answer to two decimal places.
- **2.** Cory is making a poster of common geometric shapes. He draws a square with a side of length  $4^3$  cm, an equilateral triangle with a height of  $\sqrt{200}$  cm, a circle with a circumference of  $8\pi$  cm, a rectangle with length  $\frac{122}{5}$  cm, and a parallelogram with base 3.14 cm.
  - a. Which of these numbers are irrational?
  - **b.** Write the numbers in this problem in order from least to greatest. Approximate  $\pi$  as 3.14.
  - **c.** Explain why 3.14 is rational, but  $\pi$  is not.



## Assessment Readiness



## **Selected Response**

- A square on a large calendar has an area of 4,220 square millimeters. Between which two integers is the length of one side of the square?
  - (A) between 20 and 21 millimeters
  - (B) between 64 and 65 millimeters
  - © between 204 and 205 millimeters
  - D between 649 and 650 millimeters
- 2. Which of the following numbers is rational but **not** an integer?
  - (A) −9
    (C) 0
  - **B** −4.3 **D** 3
- 3. Which statement is false?
  - (A) No integers are irrational numbers.
  - (B) All whole numbers are integers.
  - C All rational numbers are real numbers.
  - **D** All integers are whole numbers.
- **4.** In 2011, the population of Laos was about  $6.586 \times 10^6$  people. What is this number written in standard notation?
  - (A) 6,586 people
  - **B** 658,600 people
  - © 6,586,000 people
  - D 65,860,000 people
- 5. Which of the following is not true?

(A) 
$$\sqrt{16} + 4 > \sqrt{4} + 5$$

- **B** 4*π* > 12
- (C)  $\sqrt{18} + 2 < \frac{15}{2}$
- (D)  $6 \sqrt{35} < 0$

- **6.** Which number is between  $\sqrt{50}$  and  $\frac{5\pi}{2}$ ?
  - (A)  $\frac{22}{3}$  (C) 6 (B)  $2\sqrt{8}$  (D)  $\pi + 3$
- **7.** Which number is indicated on the number line?

(A) 
$$\pi + 4$$
  
(B)  $\frac{152}{20}$   
(C)  $\sqrt{14} + 4$   
(D)  $7.\overline{8}$ 

- **8.** Which of the following is the number  $5.03 \times 10^{-5}$  written in standard form?
  - A 503,000
  - **B** 50,300,000
  - © 0.00503
  - **D** 0.0000503
- **9.** In a recent year, about 20,700,000 passengers traveled by train in the United States. What is this number written in scientific notation?
  - (A)  $2.07 \times 10^1$  passengers
  - (B)  $2.07 \times 10^4$  passengers
  - $\bigcirc$  2.07  $\times$  10<sup>7</sup> passengers
  - (D)  $2.07 \times 10^8$  passengers
- **10.** A quarter weighs about 0.025 pounds. What is this weight written in scientific notation?
  - (A)  $2.5 \times 10^{-2}$  pounds
  - (B)  $2.5 \times 10^1$  pounds
  - $\bigcirc$  2.5  $\times$  10<sup>-1</sup> pounds
  - (D)  $2.5 \times 10^2$  pounds

	(A) $\frac{4}{9}$	$\bigcirc \frac{4}{5}$
	<b>B</b> $\frac{5}{9}$	(b) $\frac{5}{11}$
12.	What is the value of x if $x^2 = \frac{36}{81}$ ?	
	(A) $\frac{2}{3}$	$\bigcirc \frac{4}{9}$
	<b>(B)</b> $\pm \frac{2}{3}$	<b>D</b> ±4/9
13.	What is $\frac{[(9-2)^2]^4}{(4+3)^5}$	written in simplest form?
	<b>(A)</b> 7	
	<b>B</b> 21	
	© 49	
	<b>D</b> 343	
14	The total land area on Earth is about	

- **14.** The total land area on Earth is about  $6 \times 10^7$  square miles. The land area of Australia is about  $3 \times 10^6$  square miles. About how many times larger is the land area on Earth than the land area of Australia?
  - **A** 2
  - **B** 10
  - © 20
  - **D** 60
- **15.** What is the value of the expression  $8.3 \times 10^4 2.5 \times 10^3 1.9 \times 10^4$  written in scientific notation?
  - **(A)** 3.9 × 10<sup>3</sup>
  - (B)  $3.9 \times 10^4$
  - (C)  $6.15 \times 10^{3}$
  - (**D**)  $6.15 \times 10^4$
- **16.** What is the value of the expression  $(2.3 \times 10^7)(1.4 \times 10^{-2})$  written in scientific notation?
  - (A) 3.7 × 10<sup>−14</sup>
  - **B** 3.7 × 10⁵
  - (C)  $0.322 \times 10^6$
  - D 3.22 × 10<sup>5</sup>

```
17. What is the value of \sqrt[3]{64}?
```

- **A** 2
- **B** 4
- © 8
- **D** 16

## Mini-Task

- **18.** Amanda says that a human fingernail has a thickness of about  $4.2 \times 10^{-4}$  meter. Justin says that a human fingernail has a thickness of about 0.42 millimeter.
  - **a.** What is the width in meters written in standard notation?
  - **b.** Do Justin's and Amanda's measurements agree? Explain.

**c.** Explain why Justin's estimate of the thickness of a human fingernail is more appropriate than Amanda's estimate.